Neutrinos From Ultrahigh Energy Cosmic Ray Photon Propagation

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Before 1930, the discovery of β decay had caused a major problem for physicists the world over. It appeared that a neutron in the nucleus of an atom was decaying into a proton, releasing an electron during the process. The decay was calculated as being highly energetic, but the electron wasn't moving fast enough for it to be carrying the amount of energy it had to have.

In order to conserve momentum, the electron must predominantly be carrying kinetic energy when it is created in the decay, as the proton is held within a massive nucleus and so isn't moving all that greatly. Many physicists experimentally observed that the electron wasn't travelling fast enough to have all the calculated kinetic energy it should have, and concluded that the law of conservation of energy was being broken. In light of this, the idea of conservation of energy was on the brink of being cast off as incorrect.

In 1930, Wolfgang Pauli came up with a more acceptable solution. He postulated that a third particle was created during β decay, one with no charge, so the conservation of charge could be upheld, very little or no mass, so conservation of momentum was adhered to, and which carried the bulk of the decay energy away from the reaction, in order to keep conservation of energy. He named this particle the Neutrino, although now we know it as the electron anti-neutrino.

Cosmic rays are made up from high-energy charged particles originating outside our solar system, although the term is sometimes used to include solar products too. Most cosmic rays are atomic nuclei, around 89% of which are hydrogen nuclei. [1] While the highest energy cosmic rays are energetic enough to have photoproduction interactions in the microwave background, these collisions will cause energy loss that affects the spectrum of the cosmic rays. This is known as the *Greisen-Zatsepin-Kuzumin* (GZK) *cutoff*.

Not long after the publication of the original paper on the GZK cutoff it was proposed that a guaranteed number of ultrahigh energy neutrinos would be produced by the propagation of ultrahigh energy cosmic ray photons. Attempts to predict these neutrino fluxes and relate their detection to the neutrino cross-section at high energy and the mass of the W boson, which was unknown at that time, then followed.

The introduction of the cosmological evolution of cosmic ray sources by Hill and Schramm, in their papers in 1985 and 1986, allowed the use of measurements of the cosmic ray spectrum made by the Haverah Park and Fly's Eye experiments in order to determine allowed maximum and minimum normalizations for the flux of these propagation neutrinos, and also to calculate the detection rates for different types of detector. [2]

Detecting neutrinos produced during the propagation of ultra-high energy cosmic ray (UHECR) photons is very difficult. The neutrino flux peaks above 10^{17} eV, where the neutrino nucleon cross sections, a measurement of the probability of the neutrinos interacting with a particle [3], are of the order of 10^{-31} cm^2. Values for $\sigma_{\nu N}$ of this order of magnitude are large enough to make the Earth opaque, but 100-1000 km of water are still required in order to ensure an interaction will occur. Because of this there is no sensitivity to upward neutrinos and only low efficiency in detecting downward neutrinos.

Under these conditions, a large mass of water, of the order of 100 km^3, is required in order to guarantee even a few events per year. With such large volumes being used, a typical event may be assumed to be a contained one where the light flash emitted on interaction is dominated by the high energy shower associated with the neutrino interaction vertex. Special detector geometries may be employed in order to provide additional sensitivity to μ or τ leptons produced in charged current (CC) events. τ particles are produced in tau neutrino CC interactions in the material surrounding the detector and produce signals comparable in both shower energy and rate to electron neutrino CC interactions. Whether these τ interactions can be differentiated from ν_e CC interactions or not is determined by the τ energy, which affects both its decay length and energy loss, and also by the volume of the detector. [2]

Every type of neutrino interaction will generate a shower. In CC interactions between electron neutrinos and antineutrinos, the energy in the neutrinos is completely released as shower particles, with the hadronic component of the shower carrying a fraction, y, of the initial neutrino energy, E_ν, and the electromagnetic component carrying the remaining (1-y)Eν. Although the LPM effect will stretch out the electromagnetic shower, both components are likely to be containable in the detector volume. For muon neutrinos and antineutrinos and also for all neutral current (NC) interactions, a shower energy of yE_ν is used.

Showers generated by UHE neutrinos could also be detected by their radio emissions. As each interaction occurs and a shower is produced, some of the neutrino energy is emitted as photons with wavelengths in the radio wave section of the spectrum. Prototype experiments to detect this are in operation and there are plans for full experiments with a threshold of 1018 eV and effective volumes of 10^2-10^4 km^3. These experiments would be able to take advantage of the increased shower rates corresponding to the maximum neutrino flux, as shown in Figure 4.

By using Monte Carlo simulations, a calculation which involves random sampling of data [3], of the histories of individual particles in the cosmic background radiation and including energy-loss processes such as photoproduction, adiabatic losses and production of electron-positron pairs in the local universe and extending the model to cosmological distances, a detailed method for calculating neutrino production from proton propagation can be made. It is an advantage to use the SOPHIA event generator in this instance, as it gives detailed simulations of the proton/neutron interactions with cosmic microwave background protons. [2] For a detailed explanation of the SOPHIA event generator, see Ref. [4].

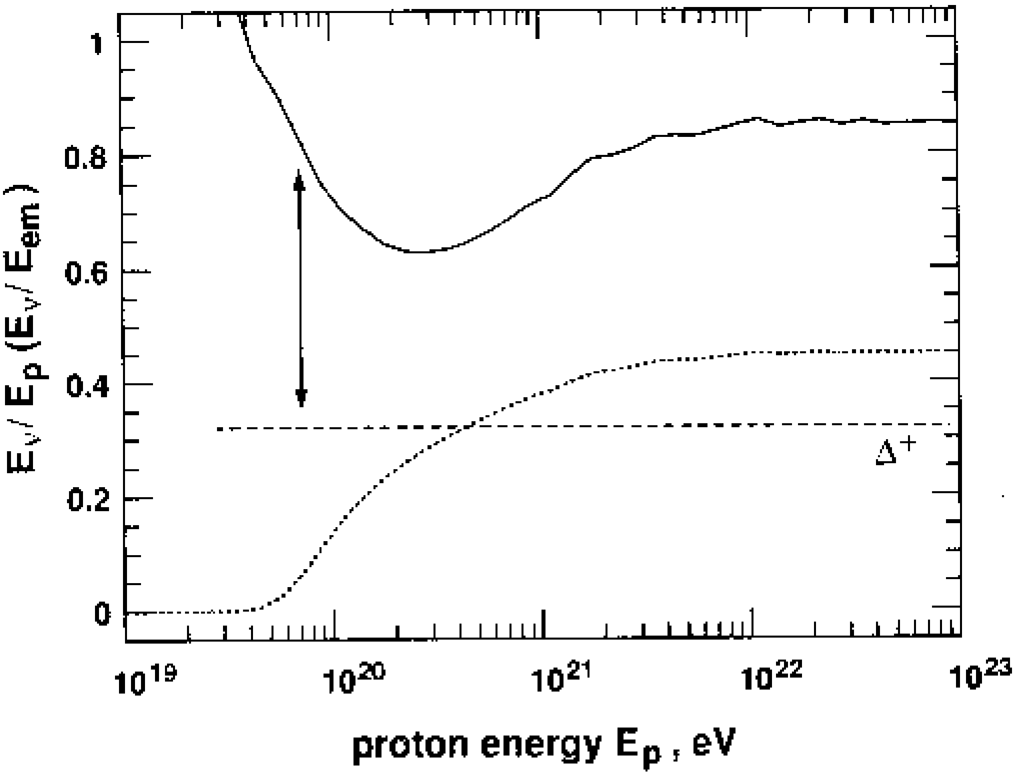

Figure 1 Neutrino production efficiency, summed over flavours, as a function of the injection energy of the protons. [2]

The results of the Monte Carlo simulations have been presented in Figure 1, where the solid curve is representative of the ratio of the energy carried by the neutrinos to the energy carried by electromagnetic particles caused by photoproduction in 200 Mpc cascades. The broken is the same ratio, only for Δ^+ resonance approximation while the dotted curve shows the total neutrino energy relative to the injected proton energy. [2]

For the overall yield of neutrinos (the number of neutrinos produced per proton), the dominant feature is at $E_p \approx 5\times10^{19}$ eV, where the GZK process activates. The ratio in radiative energy between neutrino energy and yield is very much dependant on the ratio of charged to neutral pion production. [2]

If pion production occurred only through Δ^+ resonance, the ratio would be something to the order of 1/3, where approximately ¾ of the energy goes to neutrinos for charged pions. At high energies, isospin “democracy” suggests the ratio should tend to 1. With low energy protons, direct production of charged pions again increases the neutrino yield above the expectation level from Δ^+ resonance. [2]

Now we must place this production model into an astrophysical setting and using a power law with a high-energy exponential cutoff, shown as Equation (1), for the proton production source spectra.

$$\frac{dN}{dE} \alpha E^{-\alpha} x \exp\left[\frac{-E}{E_c}\right] \qquad (1)$$

α=2 for Equation (1) unless otherwise stated and E_c, the cutoff energy, is $10^{21.5}$ eV. Adiabatic losses during propagation are calculated using H_o=75 km s^{-1} Mpc^{-1} and neutrino energies are redshifted by (1+z), where z is the redshift of the interaction site. [2]

Energy degradation of UHE protons in propagation in the microwave background is very fast and the mean free path for photoproduction interactions is 3.8 Mpc when a photon energy of approximately $6x10^{20}$ eV is used. Therefore protons with an energy of around 10^{21} eV interact twice or more in the first 10 Mpc of propagation and lose close to half of their injection energy in doing so, on average. Around 40% of this energy loss goes into producing neutrinos. So, as can be seen here, the neutrino flux originates from the initial stages of photon propagation. [2]

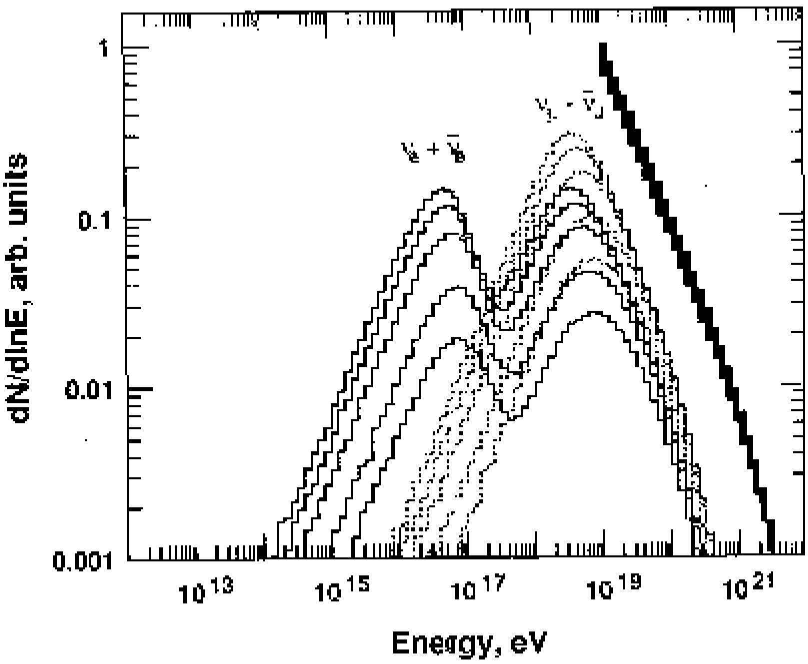

Figure 2 Neutrino fluxes produced during proton propagation over distances of 10, 20, 50, 100 and 200 Mpc, with the latter being the one at the top. The bold line is the proton injection spectrum from Equation 1. [2]

Figure 2 shows the fluxes of ν_e and ν_μ after proton propagation. Approximately 60% of the final neutrino fluxes are generated within the first 50 Mpc and over 80% in the first 100 Mpc. There are few neutrinos from the second half of the maximum propagation distance as there are no particles with energy greater than 10^{20} eV and photoproduction interactions are rare. Therefore, from the point of view of neutrino production, a source at a distance of 200 Mpc produces a fully evolved spectrum and the scenarios on a cosmological scale that are described below have neutrino yields scaled to this. [2]

As can be seen in Figure 2, muon neutrino spectra have only 1 peak, while electron neutrino spectra have 2, as the first is due to electron antineutrinos produced in neutron decay. The decay length for neutrinos is equal to the photoproduction interaction length at around 4 x 10^{20} eV and neutrons of lower energy than this are more likely to decay than to interact. The second electron neutrino peak is predominantly electron neutrinos from μ^+ decay, mixed with a small number of electron antineutrinos from neutron photoproduction. As expected, the ratio of (ν_μ + $\overline{\nu}_\mu$)/(ν_e + $\overline{\nu}_e$) in the second peak is 2, while integration over the whole spectrum shows the ratio is closer to 1. [2]

The shift of peaks to lower energies in Figure 2 is because, at longer distances, lower energy protons suffering photoproduction interactions and so generating lower energy neutrinos. As the maximum redshift of the source that is being considered here is approximately 0.05, the effect due to adiabatic losses is hardly noticeable. [2]

While it would at first be logical to assume the neutrino yield does not increase for source distances over 100 Mpc, redshift effects cause the yield to continue rising. As the minimum proton energy for photoproduction interactions decreases as $(1+z)^{-1}$, the number of interacting protons increases; therefore there is an increase in the neutrino yield even at small redshifts. [2]

For expansion of the model to cosmological distances we will focus on uniformly distributed sources with identical proton injection spectra.

Although homogeneous source distributions are not favoured by the resulting source energy requirements and arrival proton spectra, as discussed in references [5] and [6], they serve as generic models that may be rescaled to account for enhancements in the local density and also for nearby point sources. [2]

We can write the local neutrino flux of favour i, generated by the propagation of cosmic rays over cosmological distances, as an integral over redshift and the proton energy E_p^s, where ‘s’ denotes the source [2]:

$$F_i(E_{\nu_i}) = \frac{c}{4\pi E_{\nu_i}} \int\int L(z, E_p^s) Y(E_p^s, E_{\nu_i}, z) \frac{dE_p^s}{E_p^s} dz \qquad (2)$$

Where $Y(E_p^s, E_{\nu_i}, z)$ is the neutrino yield function and is calculated by

$$Y(E_p^s, E_{\nu_i}, z) = E_{\nu_i} \frac{dN_{\nu_i}}{dN_p dE_{\nu_i}} \qquad (3)$$

and $L(z, E_p^s)$ is the source function per unit redshift, calculated by

$$L(z, E_p^s) = \mathrm{H}(z)\eta(z)L_0(E_p^s) \qquad (4)$$

where H(z) is a parameter for the cosmological source evolution, η(z) describes the cosmological expansion and $L_o(E_p^s)$ is the source proton luminosity, a properly normalized version of the source spectrum described by Equation (1). [2] It is calculated by

$$L_0(E_p^s) = P_0\left(\int_{E_{\min}}^{E_{\max}} E_p^s \frac{dN_p}{dE_p^s} dE_p^s\right)^{-1} E_p^s \frac{dN_p}{dE_p^s} \qquad (5)$$

with dN_p / dE_p^s determined by Equation (1) and P_0 denoting the injection power per unit volume. η(z) is described by the equation

$$\eta(z) = \frac{1}{\mathrm{H}_0(1+z)}\left[\Omega_M(1+z)^3 + \Omega_A + (1 - \Omega_M + \Omega_\Lambda x(1+z)^2\right]^{-1/2} \qquad (6)$$

which, if an Einstein-de Siter universe (Ω_M=1,$\Omega_{□}$=0) is assumed, simplifies to

$$\eta(z) = \frac{1}{\mathrm{H}_0(1+z)^{5/2}} \qquad (7)$$

The neutrino yield function, Y, is evaluated using the Monte Carlo result for a source at a distance of 200 Mpc and the scaling relation

$$Y(E_p^s, E_\nu, z) = Y((1+z)E_p^s, (1+z)^2 E_\nu, 0) \qquad (8) \qquad [2]$$

Where E_ν is the neutrino energy. In scaling E_ν, a factor of (1+z) appears due to redshifting the neutrino energy from its observed value to its production value. E_ν and E_p^s are both scaled by (1+z) in order to keep the same invariant reaction energies in the presence of a higher cosmic background temperature.

While the use of the scaling relation makes the mathematical work easier, it has the disadvantage of introducing some approximations into the model, causing some flaws. As shown in Figure 2, neutrino production is overestimated at low redshifts (z □ 0.05). But the contribution to the total fluxes coming from z < 0.05 is very small, so this overestimation is acceptable. At high z, the ratio of neutron decay to neutron photoproduction shifts in favour of photoproduction, causing the model to predict a higher flux of electron antineutrinos around 10^{16} eV while, at high energies, the flavour distribution may be altered, but the sum of electron neutrino and antineutrino fluxes remains the same. [2]

Using the local cosmic ray energy density, a rough estimate of the injection power of cosmic rays with energy above $E_{min} = 10^{19}$ eV can be made. [7] At an energy of 10^{19} eV, the cosmic ray flux is approximately 2.5 x 10^{-28} cm^{-2} s^{-1} sr^{-1} GeV^{-1}. Assuming all cosmic rays at this energy are extragalactic, the 1019 eV cosmic ray flux is at injection and that the differential proton spectrum at injection is a power law with spectral index α = 2, the cosmic ray energy density, ρ_e, is as follows

$$\rho_e = \frac{4\pi}{c} \int E \frac{dN}{dEd\Omega\, dAdt} dE \qquad (9)$$

where dN / (dEdΩdAdt) is the cosmic ray flux. Using an energy of 10^{19} eV we find ρ_e = 1.1 x 10^{47} J Mpc^{-1} per decade of energy. In order to determine the injection power needed to maintain this energy density, an assumption must be made about the lifetime of the cosmic rays, τ_{CR}. One approach is to assume a lifetime close to the Hubble time, making τ_{CR} = 10^{10} yr. This gives a power of 1.1 x 10^{37} J Mpc^{-3} yr^{-1} per decade of energy. The total power, however, depends on the maximum energy at injection. The correct method of calculating the injection power and injection spectra is to propagate the accelerated spectra from the sources to the Earth and to fit the locally observed spectrum, but we shall not do this here as it requires assumptions about the cosmic ray source distribution plus the structure and strength of extragalactic magnetic fields, all of which are beyond the scope of this essay. [2] Instead we shall use the injection power obtained by Waxman in 1995 [8], using a more simplified yet similar method. Waxman derived P_0 to be 4.5 1.5 x 10^{37} J Mpc^{-3} yr^{-1} between 10^{19} and 10^{22} eV, with the higher maximum energy compensating for a factor of exp[-E/E_c] when compared to Waxman's result.

Finally, specifying the cosmological evolution of the cosmic ray sources, H(z), is necessary. Here we shall use the parameters from Waxman's paper:

$$H(z) = \begin{cases} (1+z)^n, & z < 1.9, \\ (1+1.9)^n, & 1.9 < z < 2.7, \\ (1+1.9)^n \exp\{ (2.7 - z) / 2.7 \}, & z > 2.7 \end{cases} \qquad (10) \quad [2,8]$$

where n = 3 describes the source evolution up to moderate values of z. A model with n = 4 is also considered, briefly, up to z = 1.9 and flat at higher redshifts. Figure 3 shows the ν_e and ν_μ fluxes obtained from this nominal choice of parameters and carrying out the integration up to z_{max} = 8. Integrating to infinity shows an increase in neutrino fluxes by around 5%. [2]

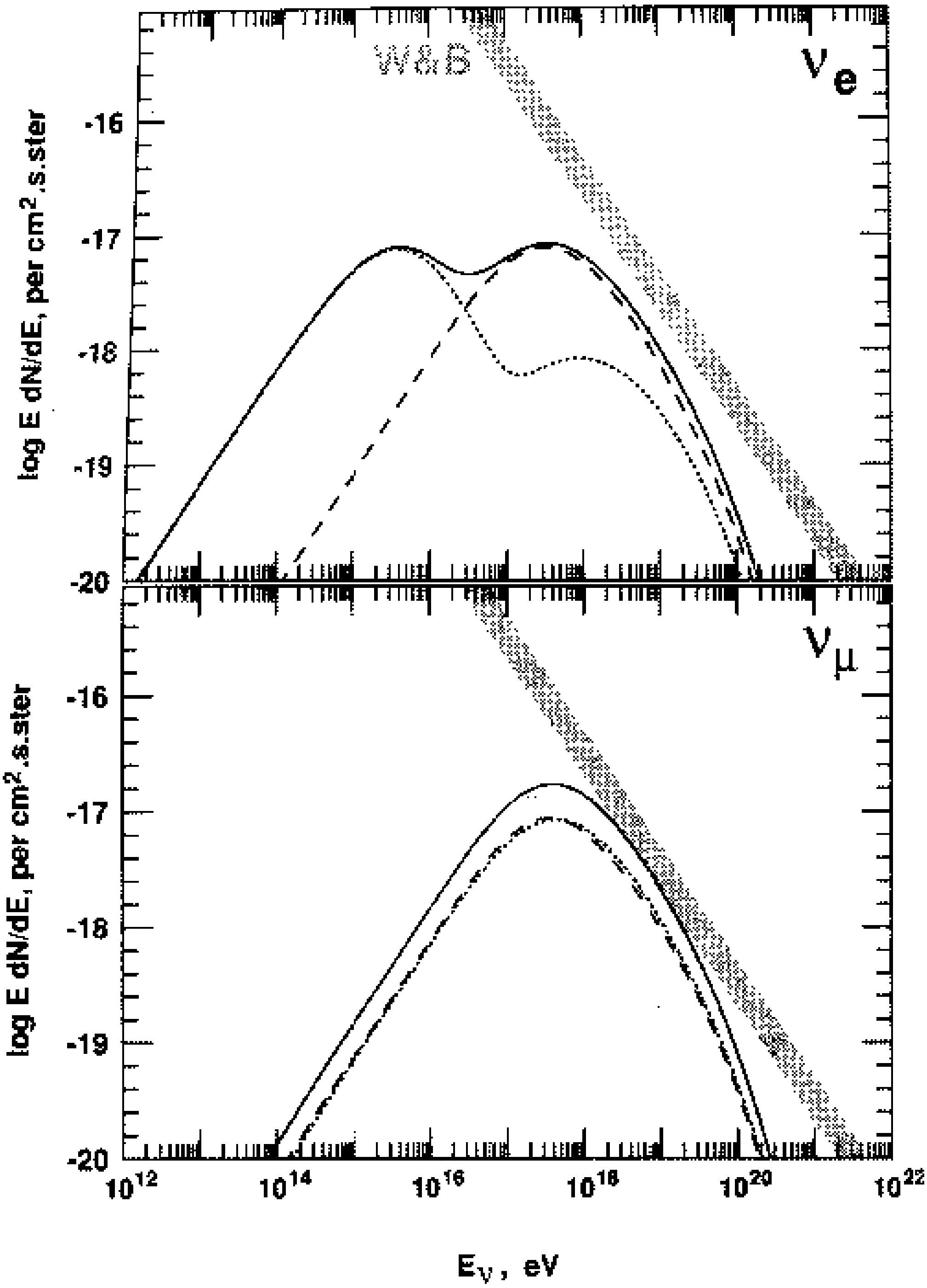

Figure 3 Top panel: electron neutrino (broken line) and antineutrino (dotted line) fluxes. Bottom panel: muon neutrino (broken line) and antineutrino (dotted line) fluxes. The solid lines are the sums of neutrinos and antineutrinos while the shaded bands are the Waxmann and Bahcall limits for neutrino production in cosmic ray sources with the same injection power. The lower edge of the band is calculated without accounting for the cosmological evolution and the upper edge is calculated with it. [2]

Figure 3 also shows the limits placed on neutrino production in cosmic ray sources as derived by Waxman and Bahcall. As their calculations used the same source evolution model as the one in this essay, plus the same injection power and similar spectra, it is useful to compare the expectation values for source and propagation neutrinos associated with astrophysical UHECRs.

As can be seen, the propagation flux calculated with the equations derived here is below the Waxman-Bahcall limit for the muon neutrino and antineutrino fluxes for energies between 10^{18} and 10^{19} eV, as a different values have been assumed for the neutrino yield per proton. Waxmann and Bahcall assumed that an energy equal to that of the injected proton is deposited into the neutrinos when deciding on their limit while here we have assumed a fraction of that energy goes into the neutrinos, as shown in Figure 1. [2]

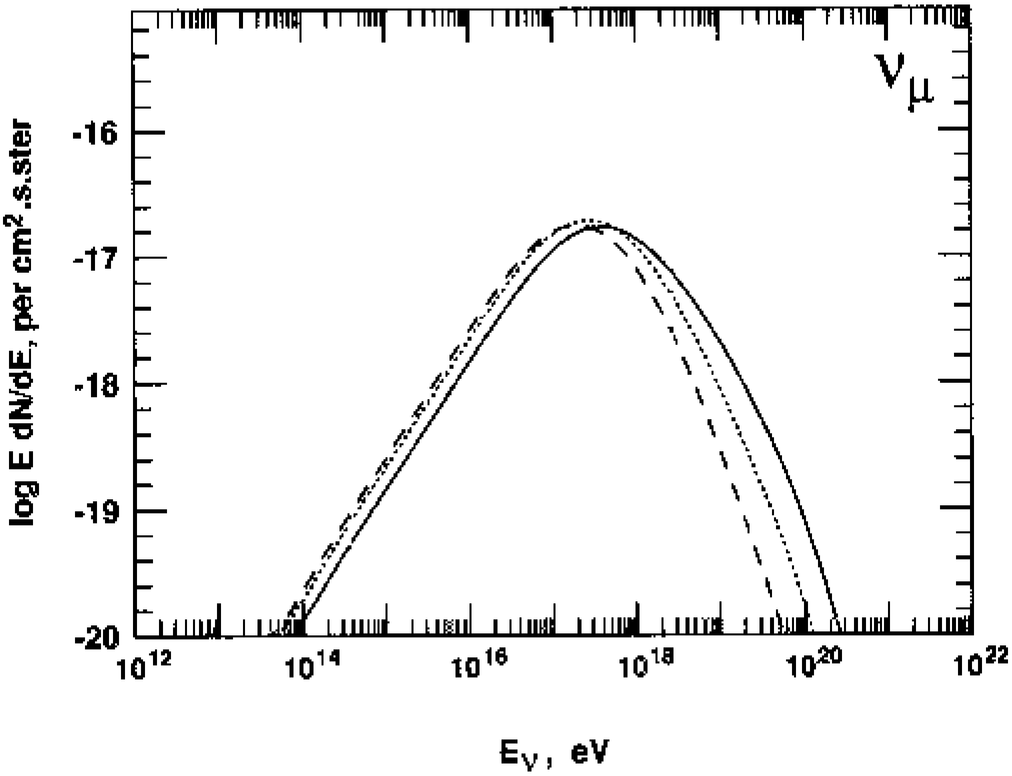

Figure 4 Neutrino flux variations resulting from differing proton injection spectra. [2]

Figure 4 shows the neutrino fluxes calculated using the source evolution of Equation (10) and differential injection spectral indices of 2.5 and 3, keeping E_c and the injection power constant. At high energies, the steeper injection spectra produce smaller neutrino fluxes, due to the far smaller number of protons with energies above 10^{20} eV that are mostly responsible for high energy neutrino production. Low energy neutrinos are generated predominately by protons injected at high redshifts, where the photoproduction threshold is decreased by a factor of (1+z). A steeper spectrum will result in an increase in the flux of low energy protons, as the total injection power is constant, and this will, in turn, result in an increase in the number of low-energy neutrinos. [2]

Our largest uncertainty when determining the magnitude of neutrino fluxes from proton propagation is related to the distribution of cosmic ray sources.

The normalization of the cosmic ray injection power and the cosmological evolution of the cosmic ray sources are both based on the assumption that the UHECRs being detected are not being accelerated by a nearby point source but are, instead, of astrophysical origin. Should the cosmic rays in fact be from a nearby point source it would have to be less than 20 Mpc away, meaning the local UHECR density would be far greater than the average density in the universe and the overall normalization of the cosmic ray power would depend on the filling factor of all the areas in the universe where the UHECR density was greater than average. [2]

Correct determination of the power in UHECRs is dependent on the strength of the extragalactic magnetic field in our local area. If we take the average strength of the field to be in excess of 1 nG we find that cosmic rays with an energy of 10^{19} eV or less will have a diffusive propagation pattern as the field pulls them in different directions by different amount, dependent on their charge. This diffusive pattern will enhance their flux on reaching the Earth, while a lower, more regular extragalactic field would guide the particles along the walls of matter concentration. However, more investigation is required before any reliable estimates about UHECR propagation can be given. [2]

Assuming homogeneously distributed astrophysical sources, a neutrino flux similar to the Waxman Bahcall limit at $E\nu > 10^{18}$ eV is obtained, meaning that, with the assumptions and restrictions described above, similar fluxes of UHE neutrinos produced in astrophysical sources and in UHECR propagation can be expected. [2]

Many kinds of air shower detector may be employed in order to observe the higher end of the neutrino spectrum, as the signature of these UHE neutrinos is showers. Because of the effective volumes required for detection are very large, the proposed OWL and EUSO air shower experiments may well be suited to observation of UHE neutrino fluxes, although with their current threshold of 5×10^{19} eV, the majority of their potential event rate would remain undetected. Their large field of view means these detectors would, in principle, be able to detect "double bang" events, caused by τ neutrinos and described by Learned and Pakvasa in reference [9], with energies above 10^{19} eV, if the threshold were lowered to that level. [2]

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